The geometry of fractals between out of measure and Artificial Intelligence (AI)

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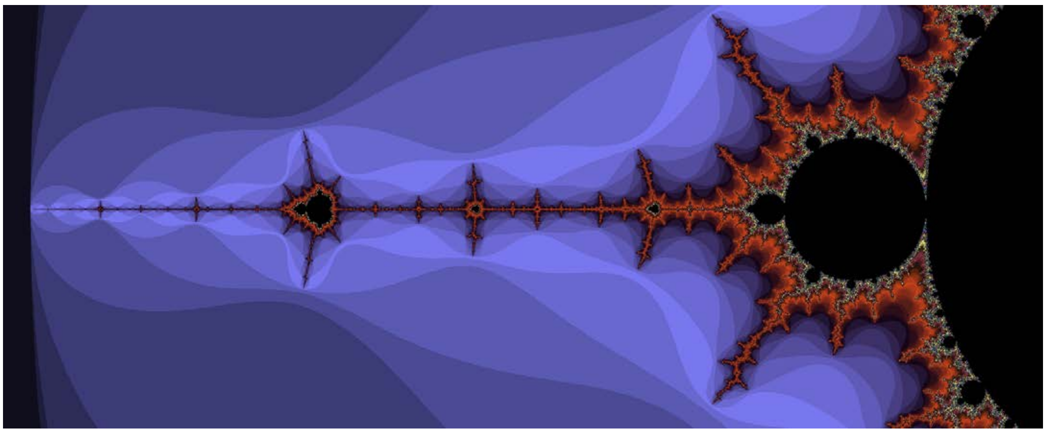
Abstract

This work intends to generate a discussion focused on fractal geometry and some of the interactions that concern it. The introduction to fractal geometries exposes the fundamental concepts of self-similarity, scale invariance and complexity. Reference is made to some well-known fractals and a representation experience relating to the Mandelbrot set is presented. The relationship between fractals and nature is subsequently described, focusing on the concepts of out of measure through references to the problem of measuring the coastline and to chaos theory in relation to the butterfly effect. The concepts of self-similarity are related to the field of architecture, describing the development considering the harmonious influence that the forms of nature and repetition produce, when taken as a model. From a graphic point of view, the most complex fractals became visible only following the development of computer graphics. In the field of Artificial Intelligence (AI), fractals find application in various fields and become an interesting experimental scenario in the field of AI text to image generators. Some examples of images generated through systems based on DALL-E 3 are proposed, relating to pure fractals, fractals in relation to architecture and fractals in relation to art. In conclusion, the theme of fractals, as a representation of the infinitely large and the infinitely small, is brought back into the ethical/philosophical debate, which involves studies on AI.

Keywords
Fractal, computer graphics, AI, out of measure, drawing.



Geometry of Fractals. Elaboration by the authors.

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Introduction

The term fractal, coined in 1975 by the Polish mathematician Benoît Mandelbrot (1924-2010), derives from the Latin *fractus*, broken, and indicates, in geometry, an object with internal homothety. The fractal is self-similar, that is, following the principle of scale invariance, it repeats more or less complex geometric shapes, so that the part appears similar to the whole. Born to describe the apparently random forms of nature, fractal geometry has proven to be applicable in various fields of scientific investigation, from engineering to physics, from mathematics to economics, up to recent use and exploration in the field of Artificial Intelligence (AI). Fractals can be generated through simple iterative processes, which, often using complex numbers, produce graphs that stand out for their intricate and infinitely detailed structures. In a way, it is possible to identify a distinction between purely geometric fractals, generated through an iterative graphic/geometric procedure, and mathematical fractals, generated through equations that find their graphic representation in the complex plane [Sala, Cappellato 2004, p. 24]. Among the best-known geometric fractals, we remember the Cantor set, the Koch curve, the triangle and the Sierpiński carpet (fig. 1); instead, among the best-known mathematical fractals we remember the Julia set and the Mandelbrot set. The Mandelbrot set, probably the most famous fractal, it is defined as the set of complex numbers c for which the function $f_c(z)=z^2+c$ does not diverge when iterated from $z=0$, that is, for a given complex number c, we start with $z=0$ and square repeatedly, adding c with each iteration. By exploring the set with increasingly higher zoom levels, we discover a surprising variety of structures, which are repeated and which reproduce, on an increasingly smaller scale, the initial shape of the entire set. A graphic representation has been created that shows successive zooms performed on the edges of the Mandelbrot set (fig. 2). Above is the set drawn as a whole, below, through a sequence created at increasing magnifications, it is possible to appreciate the principles of scale invariance, self-similarity and complexity, typical of fractals. For the generation of the set and the rendering of the visualizations, 64 bit Ultra Fractal 6.05 was used, a software specialized in the drawing of fractals. It was chosen to present the Mandelbrot set according to a monochromatic representation [1]. White indicates the points of the complex plane belonging to the set, in which the function is limited, black indicates the points do not belong to the plane, because the function diverges to infinity.

Fractals and nature

In nature we find shapes attributable to fractal geometric patterns, in the leaves of many plants, in the branches of trees, in flowers, in vegetables, in some shells, in geological formations, in the profiles of mountains and coastlines, in lightning and even in clouds. A precise idea of what a fractal out of measure means, is provided to us by the measurement of

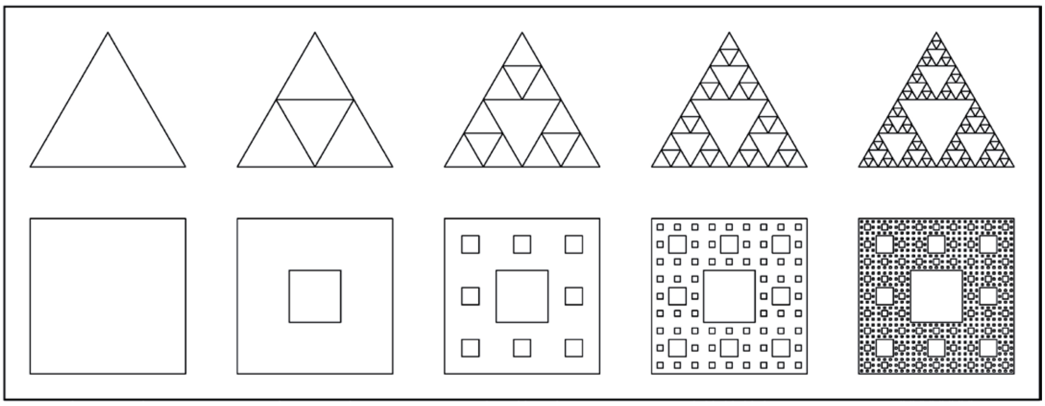

Fig. 1. The triangle and the Sierpiński carpet. Elaboration by the authors.

coastlines [2]. In reality, an operation that at first sight seems banal and intuitive, turns out to be complex and ultimately unsolvable. The length of the coast varies depending on the scale at which it is measured. When measured on a larger scale, considering the large bays and inlets, the total length of the coast seems relatively short. However, when measured at a smaller scale, including small rocks and coastal details, the length increases significantly.
From a geometric point of view, the phenomenon can be illustrated, for example, through the properties of the Koch curve (fig. 3).
The curve is constructed in the following way: 1) draw a flat segment; 2) it is divided into three equal parts; 3) the central segment is replaced with a cusp formed by two other segments of length 1/3 of the starting segment; 4) the procedure is repeated infinitely on each segment. By iterating the process n times, we obtain an open broken line with an origin point, an endpoint, and an infinite length. This infinite dimension, enclosed within an apparently finite geometric figure, leads us to the concept of chaos, understood in the etymological definition of χάος, chasm and abyss [3].
The concept of chaos holds a fundamental role within the fractal world. The out of measure has a very close relationship with chaos, indeed some natural phenomena present themselves as chaotic when faced with the attempt to be traced back to very precise and

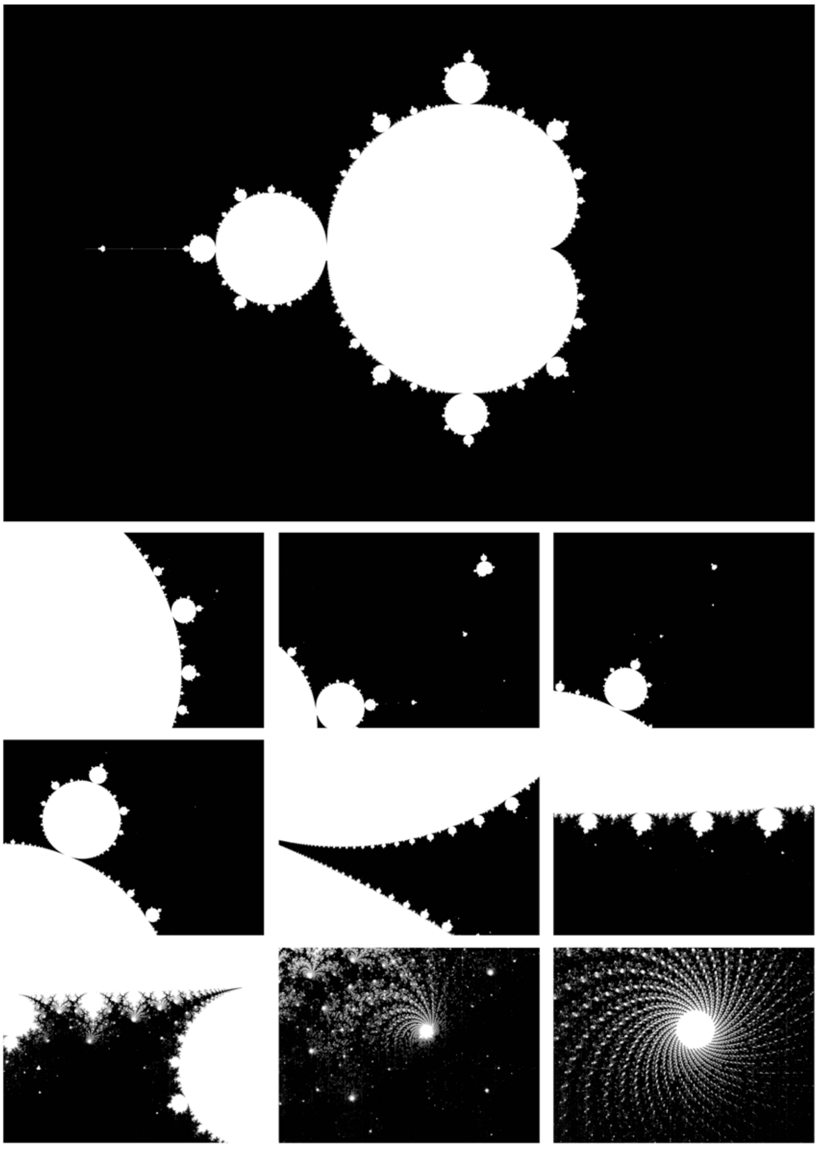

Fig. 2. Exploring the Mandelbrot set. Elaboration by the authors.

determined laws. Among the natural facts that mostly fall into this category we find meteorological phenomena, fluid-like behaviors for which it does not appear possible to precisely determine clear and reliable developments, due to the countless and very small conditions that would enter into the calculation. The butterfly effect is a term through which the concept of sensitivity to initial conditions was outlined. It is an effect by which a small change in one state of a deterministic nonlinear system can result in large differences in a later state. The butterfly effect was first analyzed by Edward Lorenz (1917-2008) in 1962, published in a 1963 paper [Lorenz 1963] and became popular with this name following a conference entitled *Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?*. Lorenz discovered this effect using a meteorological model, and noting that a small rounding of the input data produced a large change in the output data. This property can be expressed graphically through mathematical tools such as the Lorenz attractor.
Fractal phenomena in nature have been glimpsed in the astronomical field, for example in the distribution patterns of galaxies and stars, in the formation of snowflakes, in the behaviour and design of microscopic structures. In this sense, fractal geometry seems to have an irreducible behaviour, and a definition that oscillates between an ordering criterion and the very definition of out of measure and chaos.

Fractals, architecture and computer graphics

The concept of self-similarity is evident in many architectural works. Various studies have been carried out with the aim of giving this precise meaning, from the urban scale up to the architectural detail and the design object. The principle should remain unchanged: a building,

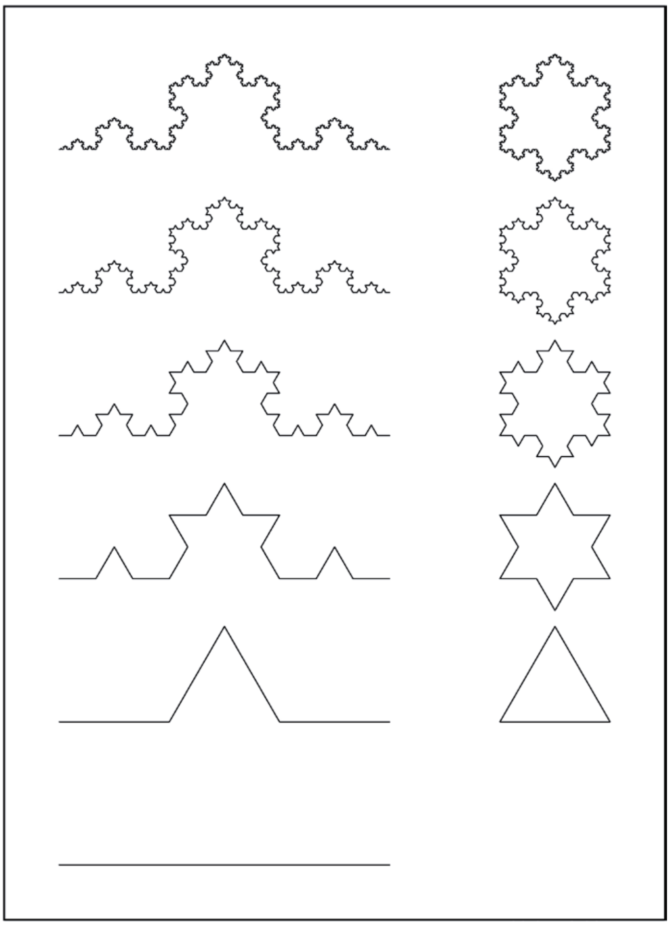

Fig. 3. The Koch curve and snowflake. Elaboration by the authors.

a plan, an object is similar to a part of it. This is not always found in the innumerable quantity of works assimilated to fractals, but it emerges clearly and evidently in others. Similar characteristics to fractal geometry are visible e.g. when observing Hindu temples [4]. These sacred places are a condensation of symbolism, an expression of the symbiotic relationship between devotee and divinity, the consequent drawing and design of order and geometric correspondences. As we see in figure (fig. 4), the column reflects the geometry of the plan, expressing the same organizational quality of thought and project. If this is visible from a

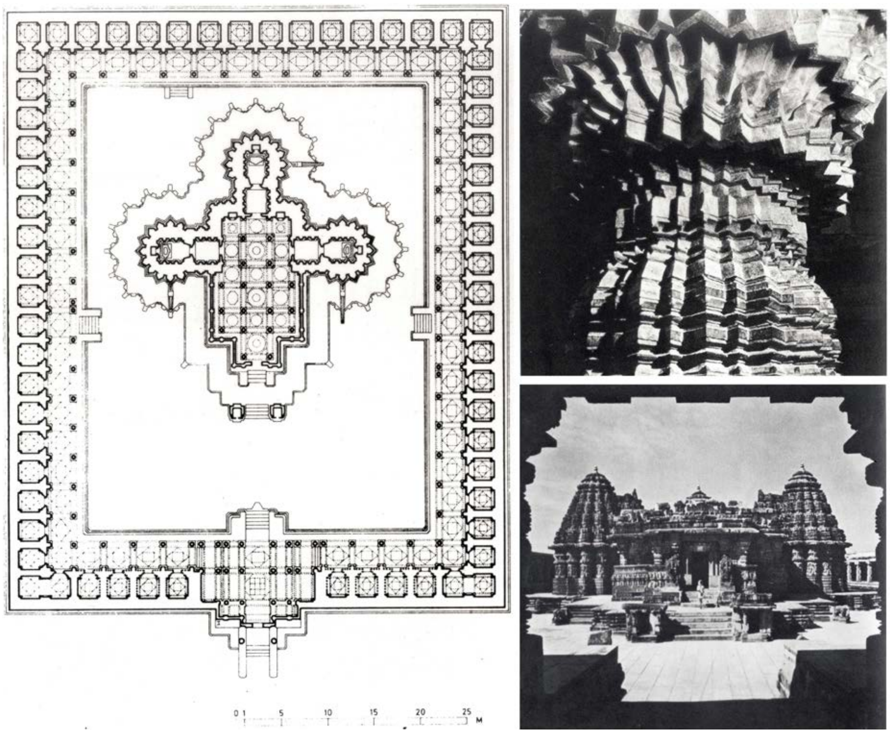

Fig. 4. Plan, detail and view from the entrance of the Somnathpur temple, built in 1268, India (from: Benevolo 1988, p. 138).

glance at the graphic works, it is also true that even the progressive reduction in scale relative to similar forms on the facade, suggests to the observer a certain familiarity with the object of observation. This, in some way, leads us to consider the harmonious influence that the forms of nature contain and produce when taken as a model [5].
An important attraction factor is determined by repetition. This leads to explaining how fractal figures described at the beginning of the 20th century, such as the Sierpiński triangle, find correspondence in earlier art and mosaic works [Conversano, Tedeschini 2011].
One of the limits of fractals was that they were confined for a long time to a purely theoretical field, made up of formulas intended for the interest of a few mathematicians [6]. Fractals, as equations, were not fully visible until the advent of computer graphics. The emergence of computer graphics and its diffusion to the public since the 1980s has given new life to Mandelbrot research, allowing previously unthinkable processing and graphics to be produced [7]. Fractals have been visualized and calculated with various computer graphics software; however, it must be kept in mind that the graphic visualization of fractals always involves a simplification, as their indefinite complexity is simplified by the number of finite iterations.
Among the forgotten software, intended for the general public, we remember Autodesk Chaos, a software package released in 1990 that included the possibility of calculating, manipulating and visualizing different types of fractal objects (fig. 5).
Over time, countless software and plugins have been developed for the generation and visualization of fractal objects, useful for the development of artistic works, but also for modelling realistic objects such as trees, territories and other fractal geometries.

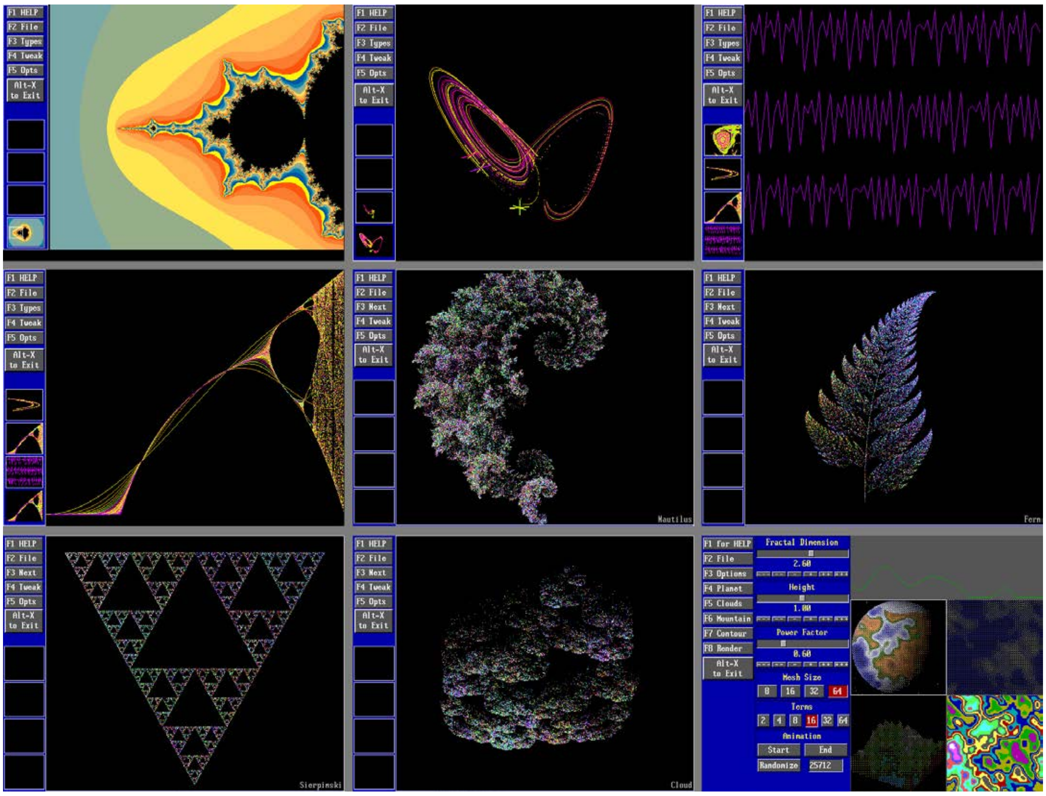

Fig. 5. Fractal geometries processed in Autodesk Chaos. Elaboration by the authors.

Fractals and AI

In the field of Artificial Intelligence (AI), the principles of fractal geometry can be used in several ways. Fractals are useful, for example, in image compression, using the principles of self-similarity to obtain high compression ratios; they can be used to perform analysis techniques in machine learning (ML) and deep learning (DL); they can be useful in developing predictive and simulation models in various domains. From the point of view of graphic disciplines, an interesting field in which fractal principles can be used, is that relating to image generation. The Generative Adversarial Networks (GANs) have emerged as tools for generating realistic images.
By training a GAN on a dataset of fractal images, the generator network learns to produce new images that resemble the input fractals, ensuring that the images generated exhibit typical characteristics of fractals such as self-similar patterns and complexity. This approach allows for the creation of new and visually interesting pseudo-fractal images that may not exist in nature.
Another type of generative models are the Variational Autoencoders (VAEs). Learning a latent representation of the input data, VAEs offer a probabilistic framework for generating fractals, allowing for exploration of the space to create variations of existing fractal patterns. For the optimization of the generative processes of fractal images, Reinforcement Learning (RL) and Transfer Learning (TL) techniques can be used. Referring again to AI techniques used for image processing, we see how the use of Neural Style Transfer (NST) algorithms, a technique which involves the application of the artistic style of an image to another image while preserving its content, makes it possible to produce singular fractal patterns with great visual impact. The development of AI Text-to-Image and Text-to-Video generators [8] allows the production of fractal images useful both for artistic purposes and to support the search for design ideas. With the aim of testing the results related to textual inputs containing explicit references to fractals [9], in text-to-image Artificial Intelligence systems based on DALL-E 3, three different solutions were explored. The image (fig. 6) shows the results for

Fig. 6. The Mandelbrot set and the Koch curve according to AI. Elaboration by the authors.

two specific requests: the four images above express an AI interpretation of the Mandelbrot fractal, while the four images below show an interpretation of the Koch curve.
As shown in the image (fig. 7), the AI's responses to requests relating to architectures inspired by fractal geometry. At the top the reference is to realistic public buildings, at the bottom to conceptual architecture inspired by three-dimensional fractals (Sierpiński tetrahedron and mandelbulb).

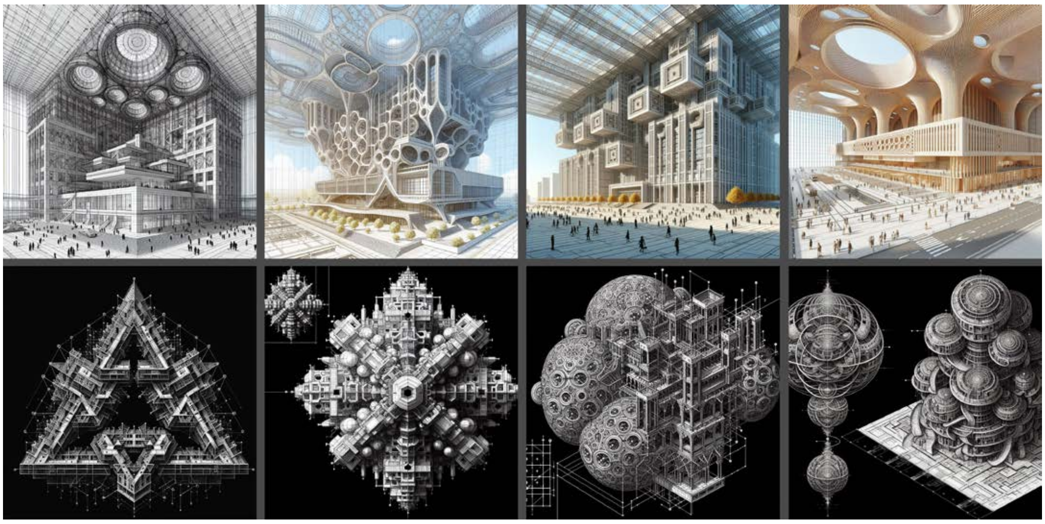

Fig. 7. AI's responses about architectures inspired by fractal geometry. Elaboration by the authors.

The image (fig. 8) proposes outputs related to artistic abstractions. Above the request was associated with the representation of a mosaic inspired by the triangle of Sierpiński; below the results are related to a generic request regarding fractal artwork. The images produced through the use of AI systems summarize requests linked by a common thread: the keyword fractal. The first group, in which the request was directed precisely towards the production of images relating to two known fractal sets, did not produce a correct outcome.
The images, although apparently presenting fractal characteristics, appear to be stereotyped and ultimately not responsive to the request (they are geometrically incorrect). The images belonging to the second and third groups, however, appear to be more interesting, correctly representing a reference to fractal forms in the architectural and artistic fields.

Fig. 8. AI's outputs related to artistic abstractions. Elaboration by the authors.

Conclusions

The 'fractal universe', understood not only as an abstract mathematical place, but also as a tool for understanding part of existence, graphically reveals the relationship between macrocosm and microcosm, visually reporting complementary relationships between the large and small scale. Out of measure and chaos seem to be two irreducible characteristics of nature. From the infinitely large to the infinitely small, the richness of details that characterize every moment of our life requires the use of a selection filter, this filter is represented by the act of establishing a reference unit, a metric scale within which the rest can be related and compared. The proliferation of numerical data that seems to escape the control of the human mind re-proposes the dichotomy that exists between sectionalization, extreme specialization and a holistic and unitary approach to things. This opens the way to debates that fall more into the ethical/philosophical field, rather than the practical/applicative one. The ethical theme is widely explored in the field of Artificial Intelligence, and new technology more generally. If Federico Faggin, inventor of the microprocessor, theorizes the irreducibility of human consciousness to electrochemical mechanisms [Faggin 2022], Luciano Floridi invites us to understand that AI is not a new form of intelligence but rather a new form of ability to act [Floridi 2021]. From the point of view of architecture and geometry, the association between fractals and artificial intelligence can allow us to explore unconventional and original graphic solutions, also encouraging the production of three-dimensional models starting from sketches. Fractal geometry and artificial intelligence can also cooperate towards finding architectural solutions that improve the efficiency and sustainability of buildings. However, despite these practical implications, from a more theoretical and general point of view, Artificial Intelligence shows its limits when faced with concepts such as measure and out of measure, intrinsically present in fractal geometry, this extension being, in the writer's opinion, the prerogative of the human mind.

Notes

[1] The well-known multicolored images are generated by coloring the points not belonging to the set, depending on how quickly the sequence varies to infinity.

[2] Regarding the problem, see: Mandelbrot 1967; Mandelbrot 1975, pp. 21-42.

[3] To be considered together, in opposition or rather in harmony, with the concept of κόσμος, ordered universe.

[4] Regarding the role of fractal geometry in Indian Hindu temple architecture see: Sardar, Kulkarni 2015.

[5] For further information on this topic see: Taylor 2007.

[6] This is especially true for complex mathematical fractals, which cannot be represented by manual drawing.

[7] Mandelbrot himself first displayed his set in 1980, at IBM's Thomas J. Watson Research Center in Yorktown Heights, New York.

[8] For a relatively recent overview of AI Text-to-Image and AI Text-to-Video Generators, see: Singh 2023.

[9] Using the fractal keyword in the textual descriptions.

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