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Domical Vaults in the Cistercian Abbey of Abbadia Cerreto: a Geometric Study

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Abstract

The Cistercian Abbey of Saints Peter and Paul of Abbadia Cerreto is a small jewel of Romanesque architecture hidden in the Lodi countryside in Lombardy. Both the church and the monastery (the latter now almost completely disappeared) conform to the main features of Cistercian architecture. The church is characterised by domical cross vaults, a type of cross vaulting with a raised mount common in Lombard Romanesque architecture from the 11th century to the 15th century. The research focuses on the geometric analysis of its vaults, exploiting the 3D model obtained with advanced digital survey technologies (static and dynamic laser scanning). The research represents a first step in identifying the existing links between the construction practice, geometry and architectural form of these particular types of vaults. The aim is to examine, within the framework of the general scheme characteristic of cross vaults with raised mounts, the formal characteristics of the domical ones in this case study. Future research activities aim to compare the same geometric aspects identified with those of the vaults characterising Romanesque-Lombard churches and Cistercian abbeys. The aim, indeed, is to exploit the geometric study of the vaults based on the digital model for the understanding and interpretation of the stylistic and historical-architectural aspects of the monument.

Keywords
Cistercian abbeys, lombard Romanesque, domical cross-vault, digital survey, geometric analyses.

Panoramic image taken at the entrance of the Saints Peter and Paul Church of the Cistercian Abbey in Abbadia Cerreto (photo by F. Fiorillo).

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Introduction

The Cistercian Abbey of Saints Peter and Paul of Abbadia Cerreto (fig. 1) is a small jewel of Romanesque architecture hidden in the Lodi countryside, approximately fifty kilometres away from Milan in Italy. Both the church and the monastery (the latter now almost completely disappeared) conform to the main features (spatial and morphological) of Cistercian architecture, imposed by the rule of St. Bernard and harmonised with those of Lombard architecture already present. All that remains today of the ancient monastery complex is the former abbey church and traces of the cloister arches and the *calefactorium* with ribbed ceiling and traces of frescoes.

Fig. 1. Saints Peter and Paul Church of the Cistercian Abbey in Abbadia Cerreto (photo by F. Fiorillo).

In Lombard Romanesque religious architecture, brick vaults almost always rose on square bays, providing lateral support in the form of sloping buttresses, sometimes hidden under the roofs of the naves and sometimes visible from the outside.
The interior effect (cover figure; fig. 2) of Lombard churches is that of a series of bays, separated by rounded transverse arches, each covered by a domical vault. This vault is commonly known as Lombard due to its historical and extensive application in this northern region of Italy. They are generally defined by a domed shape, with the keystones of each vault elevated above the apex of the transverse arches. As a rule, each bay in the nave corresponds to two bays in the side aisles (see below fig. 4), separated from the nave by a wall with two arches supported by massive pillars [Bradford Smith 2022, pp. 540-549].
Indeed, the specific geographical characteristics of Lombard Romanesque, which developed in a Lombardy that was larger than the present one, and the activity of the medieval *comacini* masters from the end of the 11th century and throughout the 12th century saw very extensive use of the cross vault, ribbed or not, in stone and especially in brick, mainly in religious architecture. Indeed, the geometry of the cross vault ensures continuity between adjacent vaulted spaces, which are repeated as a modular unit that determines the configuration of the interior space of the Romanesque church.
The research presented here focuses on the geometric analysis of the raised cross vaults of the Cerreto church, exploiting the 3D model obtained with advanced digital survey technologies. The generated key vertical sections of the nave highlight the lowering of the arrow of the perimeter arches compared to that of the diagonal arches, which characterise the raised mount vaults. In addition, horizontal sections with a constant pitch of the vault show the layout, confirming the geometry of the domical form. Finally, the shape and size of the wall arches and diagonal arches of a nave bay were verified. The analyses were conducted regarding the surface of the intrados, which presents a clearer and more readable geometry.

Case study: Cerreto cistercian abbey

The abbey, dedicated to St. Peter and Paul, St. Mary and St. Nicholas, was founded by the Benedictines in 1084 but was entirely replaced in 1136 by a new one led by Cistercian monks, built according to the rule of St. Bernard (Gemelli 2015). In 1139, the monastery came under the abbey of Chiaravalle Milanese. The church, located on the north side of the abbey, was probably built between 1140 [Porter 1917] or 1160 and 1170 [Fraccaro De Longhi 1958]. The floor plan presents a three-nave layout with a straight-ended apse and protruding transepts, with three open chapels on each side (six total) (see below fig . 4). A lantern tower with an octagonal cross-section stands on the crossing span (fig. 2). Cross vaults cover the naves and transepts; instead, the apse and chapels are covered by barrel vaults. The vaults of the central nave and transepts have toric brick ribbed vaults. In contrast, the ribs of the crossing bay, with a lantern tower above, were removed during 20th-century restoration.
The architectural structures of the Cistercian complex survived until their demolition in the 19th century, which only spared the church and the western arm of the cloister (already converted into a residence in the 17th century). A restoration campaign was initiated in 1895 to consolidate the church's structure and remove post-medieval additions. The restorations were conducted under the supervision of Luca Beltrami. A second restoration campaign involved the Romanesque-style renovation of the façade, conducted by the parish priest of Cerreto in the 1940s (without the approval of the superintendency).

Fig. 2. Drone photo (left) of the Cistercian Abbey and view inside the church (right) (elaboration by the authors).

Digital survey

Through an integrated digital survey approach, employing static and dynamic terrestrial laser scanners (fig. 3), it was possible to analyse the geometry of the vault in its entirety (both intrados and extrados). About 103 scans were made between the interior and exterior for the 3D acquisition of the church, employing a static laser scanner (Leica RTC360).
In contrast, a mobile laser scanner (Leica BLK2GO) was employed to digitally survey the roof space above the vaults, which is more suitable for narrow spaces that are difficult to access. Indeed, the connection to the attic spaces of the side aisles and nave is via a narrow spiral staircase (80 cm wide) contained in the polygonal tower leaning against the north transept (figs. 2, 3). The staircase provides access to the attic of the north side aisles and the attic of the transept and nave. The roof spaces of the side aisles are accessible through openings in the backbone walls, and they are connected to the west via the vestibule attic.

With the dynamic laser scanner, 12 closed paths were executed (point of arrival coinciding with point of departure) to minimise compensation errors in recording the acquired data. The first path connected the north transept and the spiral staircase. The north aisle, the narthex and the south aisle were mapped with three separate routes. Finally, to cover the area of the attic of the transept and nave, seven more routes were executed (three for the transept and lantern tower and four for the nave bays). Sufficient overlap between the routes was always ensured for the mappings' alignment.
The processing of the acquired data (registration in the same coordinate system, filtering of the scans, etc.) was followed within Leica Geosystems' proprietary software (Cyclone Register 360 PLUS). The overall point model of the abbey was then imported into the CAD environment to extract classical technical drawings such as plans (fig. 4) and sections (fig. 5) and develop the specific geometric analyses of the vaulted system.

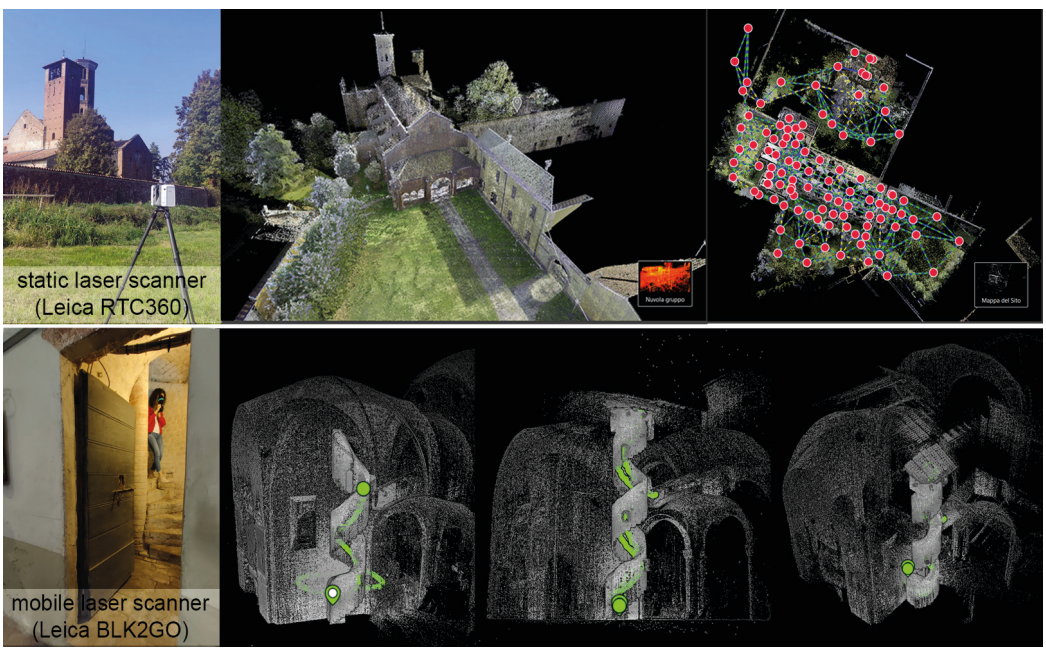

Fig. 3. Drone photo (left) of the Cistercian Abbey and view inside the church (right) (elaboration by F. Fiorillo).

Domical vaults: theoretical geometric shape

From a geometric point of view, cross vaults are obtained from the straight intersection of two equal barrel vaults. If the room they cover is square, the two directrix curves are equal, and the key generators belong to the same horizontal plane (fig. 6.1). If, on the other hand, the floor plan is rectangular, the two perimetral arches can be a circle and an ellipse so that the condition of complanarity of the upper generatrix is satisfied (vault with even-level crown).
The generatrix of the webs may be inclined instead of horizontal, in which case the cross vault is generically called a raised vault [Gaetani *et al.* 2016]. In practice, the key of the diagonal arches is higher than that of the perimetral arches. In particular, the generatrix of the two barrel vaults is inclined (fig. 6.2), giving the vault a domed appearance, typical of the vaults in Lombard architecture, used from the 11th to 15th centuries. The advantage of this 'raising' was to provide further verticality to the thrust coming from the vault and discharging onto the pillars below, thus reducing its horizontal component [Petrignani 1990]. In this case, the diagonal arches, which represent the points where the two barrel vaults intersect, are generally characterised by the presence of ribs, stone or brick ribs that may perform a load-bearing function or a purely decorative one.

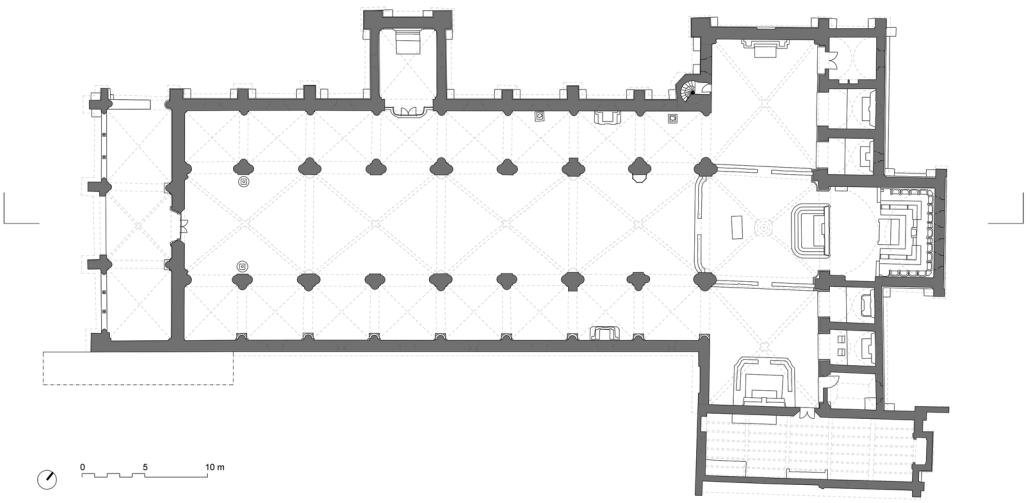

Fig. 4. Plan of the Church of the Cerreto Abbey (elaboration by the authors).

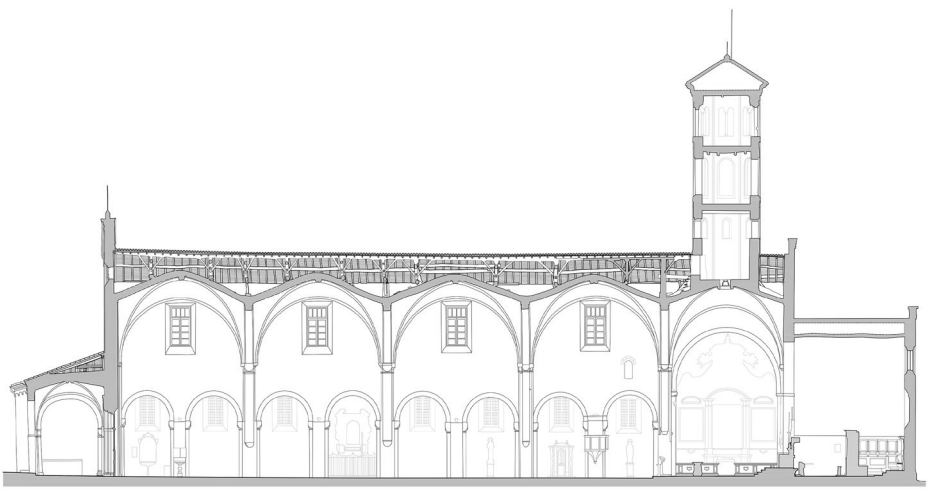

Fig. 5. Longitudinal section of the central nave of the Church of the Cerreto Abbey (elaboration by the authors).

The Roman cross vault differs from the Romanesque and Gothic cross vaults both from the point of view of its geometric shape and from the structural point of view. Generally, the Roman cross vault is constructed as an intersection of two load-bearing barrel vaults. The edges of the diagonal arches, resulting from this intersection, have a theoretically semi-elliptical shape and load-bearing function. Concentrating the thrust actions at the vertices of the plan allows large spans to be covered without the need for supporting structures in the centre and the lateral walls to become non-structural elements. On the other hand, the Gothic raised cross vault is constructed by first making the ribs, and then the webs, which are as light as possible [Docci, Migliari 1992].

Consequently, the diagonal rib, with the longest span, is elevated to a higher height than the transverse and longitudinal arches. This method offered a novel principle in vault construction, specifically the practice of designing the profile of the groin ribs and allowing the configuration of the vault surfaces to conform to them [Fletcher, Fletcher 1905].

According to Breymann [Breymann 1885], the Roman vault, rigorously cylindrical and with horizontal axes so as to ensure that the brains of the vault are at the same height as the keystone of the impost arches, evolved into the Gothic cross vault according to the following stages (fig. 7). Recognising the structural importance of the diagonal edges, in the case of large spans, these are reinforced with the construction of ribs. The elliptical shape of the diagonal arches was inconvenient in the construction of the ribs made of

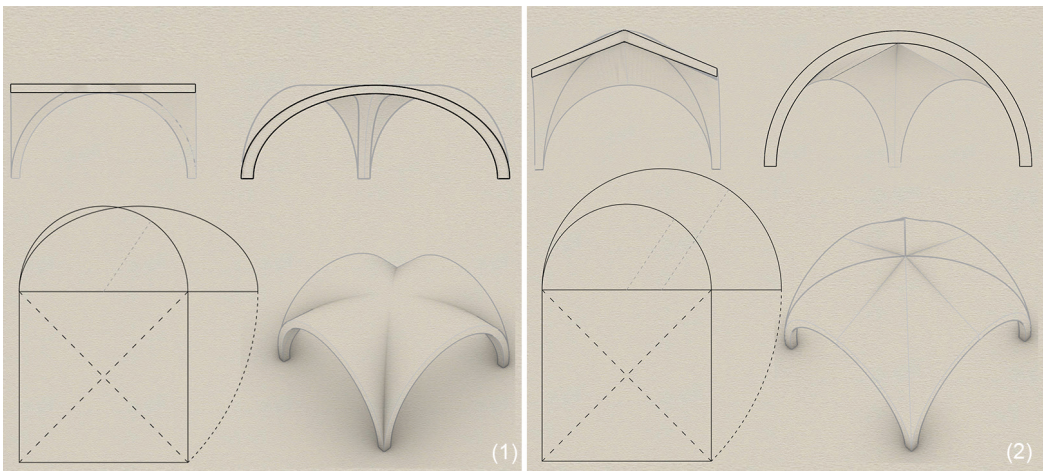

Fig. 6. 1. Theoretical model of the geometry of plain cross vault (Roman); 2. theoretical model of a type of domical cross vault (linear generatrix and diagonal round arch) (elaboration by the authors).

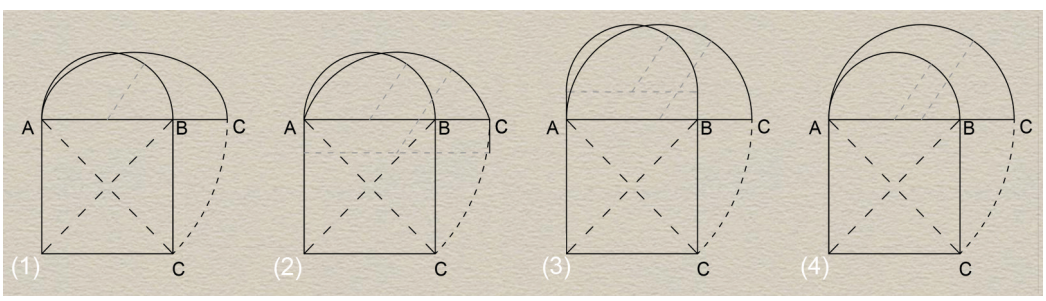

Fig. 7. Relationship between the profile of the lateral arches and that of the diagonal arches of different types of cross vaults: 1. round lateral arch and semi-elliptical diagonal arch (plain Roman cross vault); 2. round lateral arch and lowered diagonal arch; 3. raised lateral arch and round diagonal arch; 4. round lateral and diagonal (domical vault) (elaboration by the authors).

stone elements. Therefore, it was decided to realise diagonal arches with a semicircular shape, which, however, implied the need to raise the perimetral arches as well (in order to maintain the coplanarity condition of the key generators). This solution left the problem of the perimetral arches, which had to be raised to keep the vaulting constant. The pointed arch was, therefore, adopted for the perimeter arches. Finally, by renouncing the condition of coplanarity of the keystones, we arrive at the raised crown, which allowed for a more unrestricted geometric form of the cross vault [Migliari 2009].
From a purely geometric point of view, when the crown is raised, the diagonal arches are circumferential arcs, and the keystone lines are no longer horizontal but move along a straight or curved line. In this case, the longitudinal sections in the key of a domical vault sequence have a characteristic 'zig-zag' pattern, i.e. they rise and fall, due to the differences in the elevation of the keys of the parietal arches and the vault apex (fig. 5). It should be noted that the web portions of a raised crown vault sometimes have a slight uplift (the intrados surface similar to that of a spherical triangle). It is not easy, in such cases, to distinguish the cross-ribbed vault from the sail vault; the analysis must be conducted more on both a stylistic basis and a purely geometric one [Docci, Migliari 1992].
With reference to the Lombard area and the Romanesque period, local workers specialised in the construction of the so-called domical vault in the Lombard style, i.e. a cross vault with a raised crown geometrically characterised by circumferential arches for both the diagonal and perimetrical arches. This particular type of vault is also called domical precisely because of the more rounded conformation that its webs assume.
In summary, the theoretical geometrical aspects that we are going to analyse in this case study, because they characterise the domical vaults are: 1. the vault generatrix is inclined and can be a straight or curved line (this implies a zig-zag shape of the longitudinal and transversal vertical section, where the keystone of the vault is higher than the apex of the perimeter arches); 2. round arches against the walls; 3. round arches for the diagonal section.

Domical vaults: the case of Cerreto Abbey

The first analysis performed on the digital model of the Cerretto Abbey focused on the shape of the cross vaults of the nave and the transept (which are at the same level). In order to understand their geometry, the first step was to make horizontal sections with constant pitch (50 cm) of the vaults, starting from the springing line. Figure 8 shows the progressive superimposition of the profiles obtained from the point cloud model using horizontal cutting planes. This first representation immediately showed that all the vaults in question did not have the typical shape of plain cross vaults: they show a square perimeter till the crown. Instead, figure 5 shows the longitudinal sections of the nave of the Cerretto Abbey (extracted from the digital survey), which presents the typical zig-zag shape of the raised crown vaults. From this drawing, it is already possible to highlight a characteristic feature of the domical vaults studied: the generatrix initially appears as a straight line (and not an arched section), but as it approaches the centre, there is a point of inflexion, which emphasises a very domed shape.

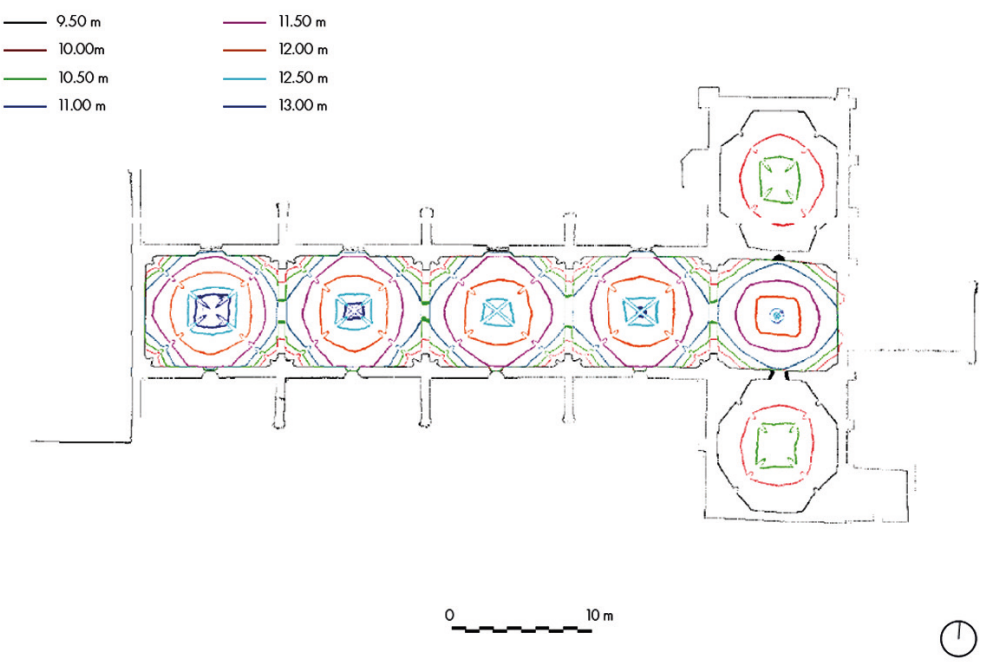

Fig. 8. Superimposition of the subsequent horizontal profiles of the vaults of the central bay of the Church of the Cerreto Abbey. (elaboration by the authors).

In order to verify the latter in detail, the fourth bay of the central nave was isolated (fig. 9). On this span, a succession of vertical sections was carried out starting from the perimeter arch to the crown at a constant pitch of 20 cm. These sections revealed several important geometric aspects. Firstly, the impost plane is constant (at approximately 8.2 m) and the lateral arch is confirmed to be round. Furthermore, it can be seen from the representation how the sections of the intrados are sharpened from the wall arch towards the keystone of the vault (from warm to cold colours). Particularly noticeable is how the generatrix visible in the central cross-section (spindle curve) does not present an arch form, nor is it perfectly straight. Indeed, the generatrix starts out straight but then shows an inflexion point from which the shape becomes more 'domical'. That is to say, as anticipated, one can see in these sections the 'spherical uplift' of the intrados of the vault near the apex, which in 3D space takes on the shape of a sort of spherical triangle. This point of inflexion, which therefore appears to be constructive, geometrically corresponds to the point of intersection in space of the two contiguous orthogonal barrel vaults. In addition, a vertical section was also made along the diagonal arch (fig. 10) that confirmed the form of the round arch and the springing line of the arch at a constant level.

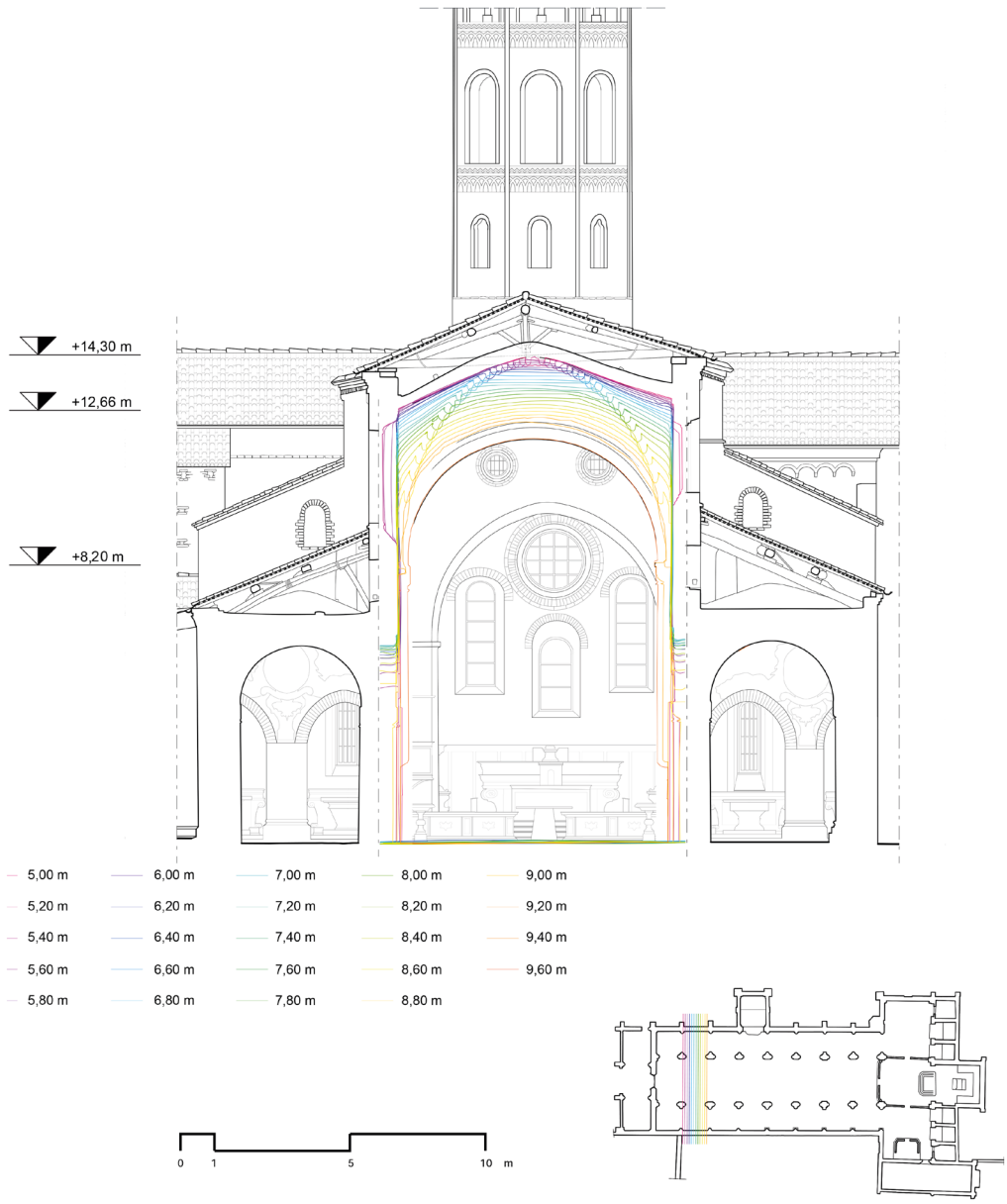

Fig. 9. Subsequent vertical cross-sections from the wall arch of one of the central bays of the church of the Cerreto Abbey (elaboration by the authors).

Conclusions

The analysis of the vaults of the Cistercian church in Abbadia Cerreto revealed the special geometry that characterises the domical cross vaults of Lombard Romanesque architecture. An accurate digital survey integrating static and dynamic terrestrial laser scanners enabled a three-dimensional observation of a typical vault using several types of sections.
The vertical section of the intrados made it possible to verify several geometric aspects of the vaults. The diagonal arch is not a horizontal straight segment, as in Roman groin vaults that present an even-level crown, but is a round arch, a choice that facilitates its execution. The perimeter arches are also round arches.
Unlike the sail vault, which many authors describe as similar to the domical vault, the generatrix is not curved but rectilinear, both in the vaults of the nave and in the smaller ones of the side aisles.
Horizontal sections likewise make it easy to recognise domical vaults from sail vaults due to the absence of circular sections. In the present case study, it has been observed that the horizontal sections tend to assume a quadrangular shape towards the top.

Fig. 10. Diagonal sections of one of the central bays of the church of the Cerreto Abbey (elaboration by the authors).

This shape corresponds to the intersection in space of the four barrel vaults starting from the perimeter arches.
This paper reports the first observations on a domical vault of the church belonging to the Cerreto Abbey. The future work is to verify systematically these analyses and geometric observations on all the vaults of this church (including those of the aisles). It is intended in the future to find the rule that determines the difference between the two types of vaults: the domical vault created with a theoretical geometric model and the one adopted by Lombard Romanesque builders, such as at Cerreto Abbey. Furthermore, it would later be possible to develop comparative work with other Romanesque churches in Lombardy.
For conservation purposes, finding the construction rule of a vault system in a historic building can also be of great use in assessing the presence of any deformations and interpreting any instability.

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