46th International Conference
of Representation Disciplines Teachers
Congress of Unione Italiana per il Disegno

Geometric Analysis of Palazzo Sormani's Vault through Drawings, Historical Manuals, and 3D Modelling

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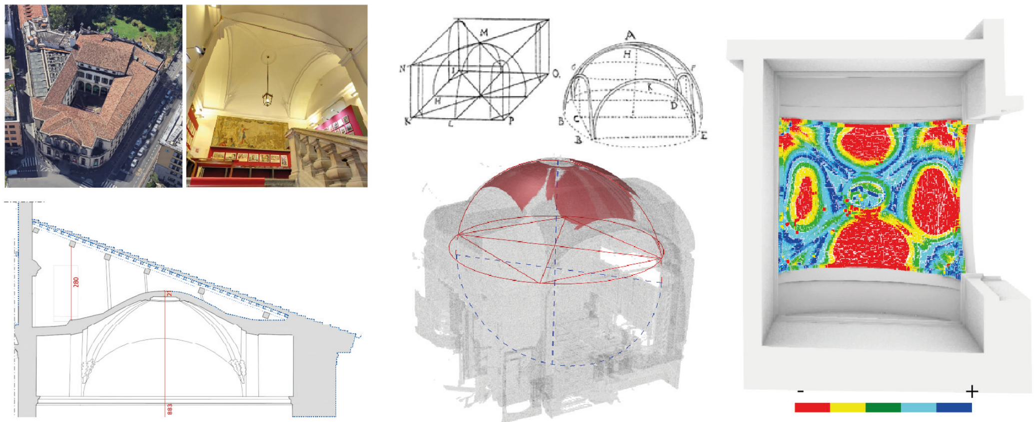
Abstract

The oldest part of Milan's public library, Palazzo Sormani, was built in the 16th century, which was followed by the intricate history of transformations and ownership switches. The research investigates the vaults designed by the architect Francesco Maria Ricchino in 1650 during the major changes of the initial building. The study examines the sail vault over the first-floor staircase landing, combining historical documentation with laser scanning to analyse its geometry and construction techniques. Historical and technical drawings, point clouds, and 3D modelling enabled a comprehensive investigation that revealed significant deviations between the ideal vault form (as described in historical manuals) and the actual structure. The discrepancy between the two can be read in the flattened vault cap and lowered parallel arches, increased on the side of the aperture toward the staircase. By comparing original drawings with manuals by Palladio, Tibaldi, and Guarini, and using digital modelling to visualise geometric anomalies, the study provides insights into Ricchino's adaptations to existing building constraints. The methodology offers a framework for investigating similar vaulted structures, highlighting potential factors behind geometric irregularities, including construction methods, settling over time, and effects from the 1945 bombing that damaged the palace. This approach enhances understanding of the architectural characteristics of vaults and further supports their preservation.

Keywords
Palazzo Sormani, sail vault, 3D modelling, geometric survey, construction history.



Palazzo Sormani in Milan, Italy. Geometrical and constructive analysis of the sail vault above the monumental staircase.

doi.org/10.3280/oa-1430-c842

Introduction

The *èkphrasis* approach applied to historical structures provides a framework for interpreting and constructing both idealised and reality-based 3D models. With the aid of historical construction manuals, this approach introduces a descriptive and imaginative discourse. In the 3D modelling context, it facilitates a dual reading: the ideal model, created by geometric logic and theoretical clarity, captures the abstract perfection of form, while the reality-based model reflects the deformations, material constraints, and constructional adaptations. This descriptive power makes the digital model not just a reproduction, but a critical reconstruction that reveals both the conceptual intention and the constructed reality of historical architecture.
Palazzo Sormani in Milan, which today houses the Municipal Library, has a complex and multifaceted construction history, spanning from the 16th century to the present day. The building's origins and the various construction phases that have marked its history, from its beginnings to the mid-18th century, have been meticulously retraced by Irene Giustina in her article *Un inedito progetto di Francesco Maria Ricchino e alcune precisazioni sulle vicende del Palazzo Monti Sormani a Milano* [Giustina, 1995]. In particular, new documentary discoveries (Fondo Monti Viani, preserved in the archives of the Archbishop's Seminary in Venegono Inferiore, Varese) have allowed for a reconsideration of architect Francesco Maria Ricchino's role in the reconfiguration of some rooms and the inner courtyard of the former palace owned by the Castaldi family. Changes made to the existing building by Ricchino, at

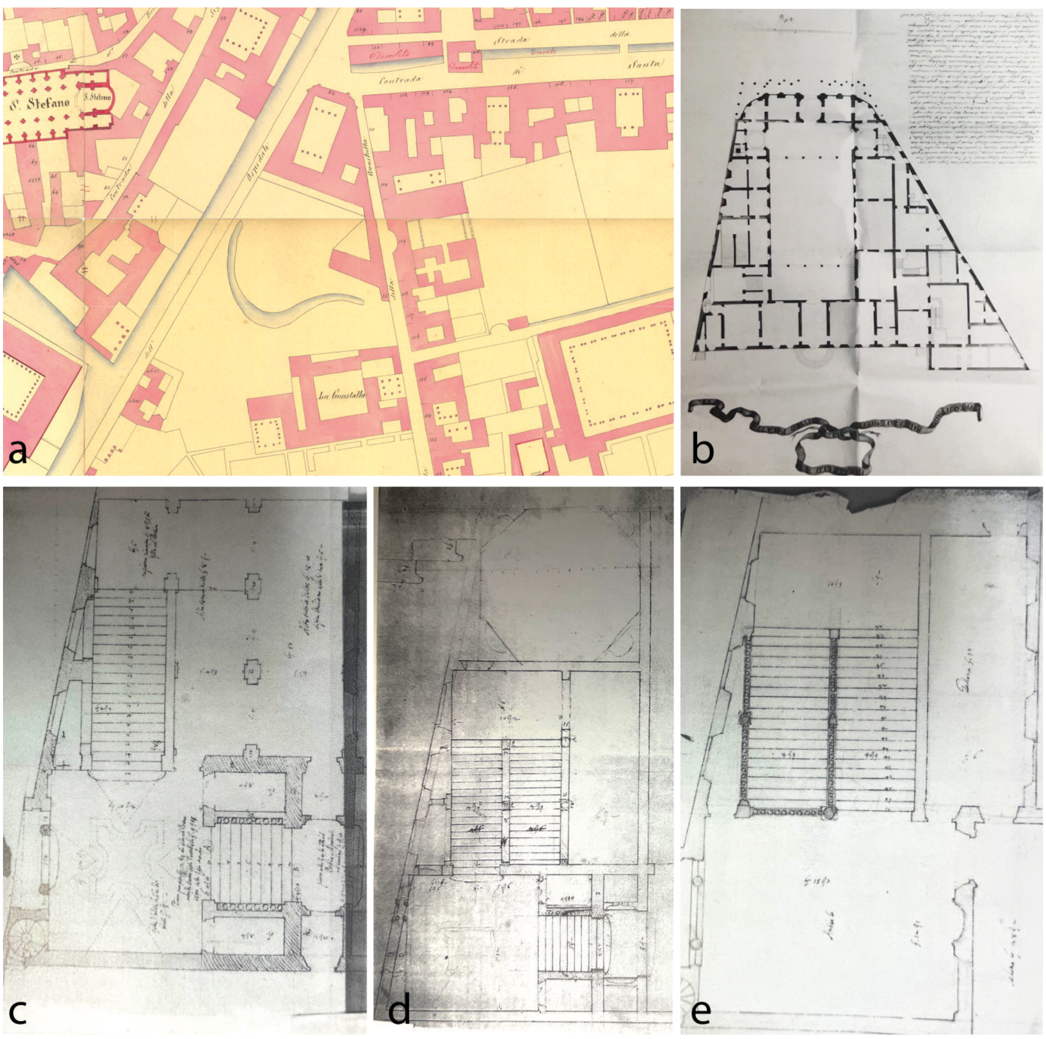


Fig. 1. a: cadastral map 1807 Astronomi di Brera, IGM (ASMi); b: drawings by F. M. Ricchino from 1640s of the vestibule and the monumental staircase published in the journal Palladio n. 16 [Giustina, 1995], ground floor; c: ground floor; d-e: first floor.

the behest of the new owner Cardinal Cesare Monti, clearly emerge after a comparison between the drawings of the actual state and the various project proposals dated 1649-1650. The patron intended to consolidate the image of his family by entrusting the project to the leading architect [Balestreri 2017, p. 28] on the Lombard construction scene at the time (fig. 1).
Following a long design process of rethinking and modifications, construction on the great monumental staircase began in 1650 [Giustina 1995, p. 54], along with the vestibule, in the west wing of the Palace, along the Naviglio canal, where the room for the Cavallerizza stables had been located on the ground floor. This was a new volume created inside pre-existing spaces, adapting the novel structures to the dimensions of the built space. The central body of the building, overlooking the rear garden, housed the large reception rooms on the first floor and other rooms on the ground floor facing the garden, both of which were to be preserved.
The insertion of the new monumental staircase required a careful survey of the existing building, as evidenced by the rich graphic documentation produced by the architect. He aimed to create a new scenic access to the noble floor that would meet the client's needs, featuring grand and spacious rooms, according to the fashion of the time and the lifestyle of the owner. The drawings published by Irene Giustina illustrate the exploration of different design solutions, both in plan and volume, with various configurations of the staircase ramps, testing their placement in different places in the building. The ultimate choice deferred from the usual configuration of the time to place it at the entrance of the palace, but it was positioned at the end of the courtyard of honour, preceded by a vestibule. The vestibule created the transitional space for welcoming guests, located between the portico facing the entrance and the two ascending ramps. Today, the landing atrium on the first floor provides access to the Sala del Grechetto.

Geometric and constructive cnalysis of the staircase and its access areas

The geometric survey activities were carried out with FaroFocus 3D laser scanner (with the scanning accuracy of approximately 1 cm) between 2021 and 2022, allowed for a three -dimensional reconstruction of the spaces of the staircase, vestibule, and atrium on the first floor, all covered by vaults of different shapes and types (fig. 2).
The staircase measures 10 × 6 m and it has two ramps (40 steps), is preceded by the vestibule, accessible from the portico via a central ramp of 10 steps. Covered by a lowered barrel vault, the staircase room has an overall height of about 13 m, of which 9.8 m is from

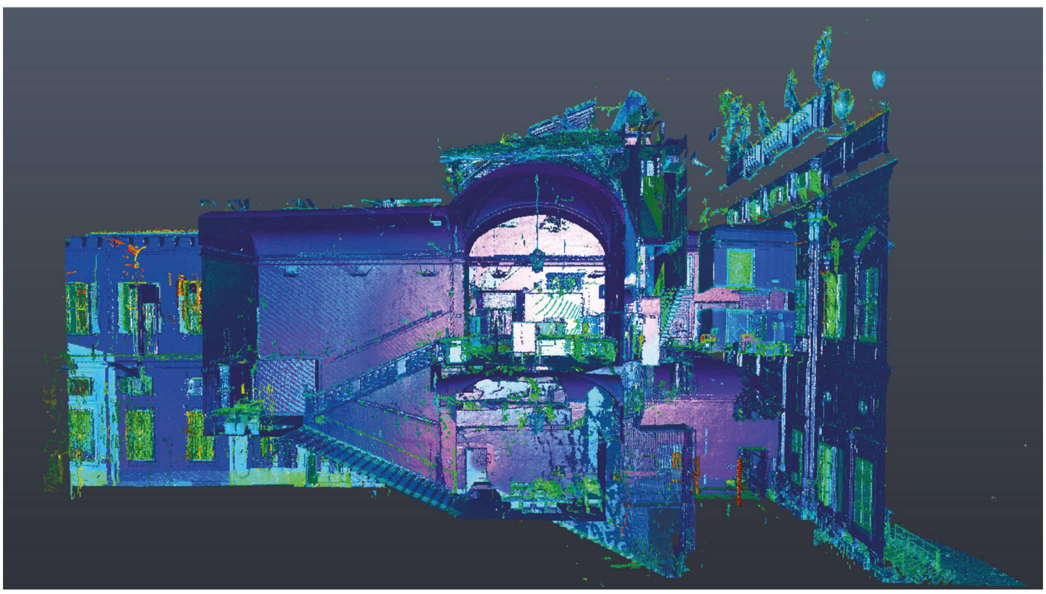


Fig. 2. Sectioned point cloud in RecapPro to show the position of the staircase and vault of the first floor (elaboration by the authors).

the mezzanine floor to the top of the vault. The situation is different for the two vaults covering the rooms on the ground floor and first floor, where the height of the vaults must necessarily have been constrained by the height of the existing rooms. In particular, the vestibule, characterised by a floor plan of approximately 5.2 x 6.4 m and a maximum height of around 6 m, features a lunette pavilion vault with lowered arches (*volta a padiglione lunettata a sesto ribassato*), with a thickness of about 30cm between the intrados and extrados. Although not highly curved, the articulated shape of the vault and the flared shape of the

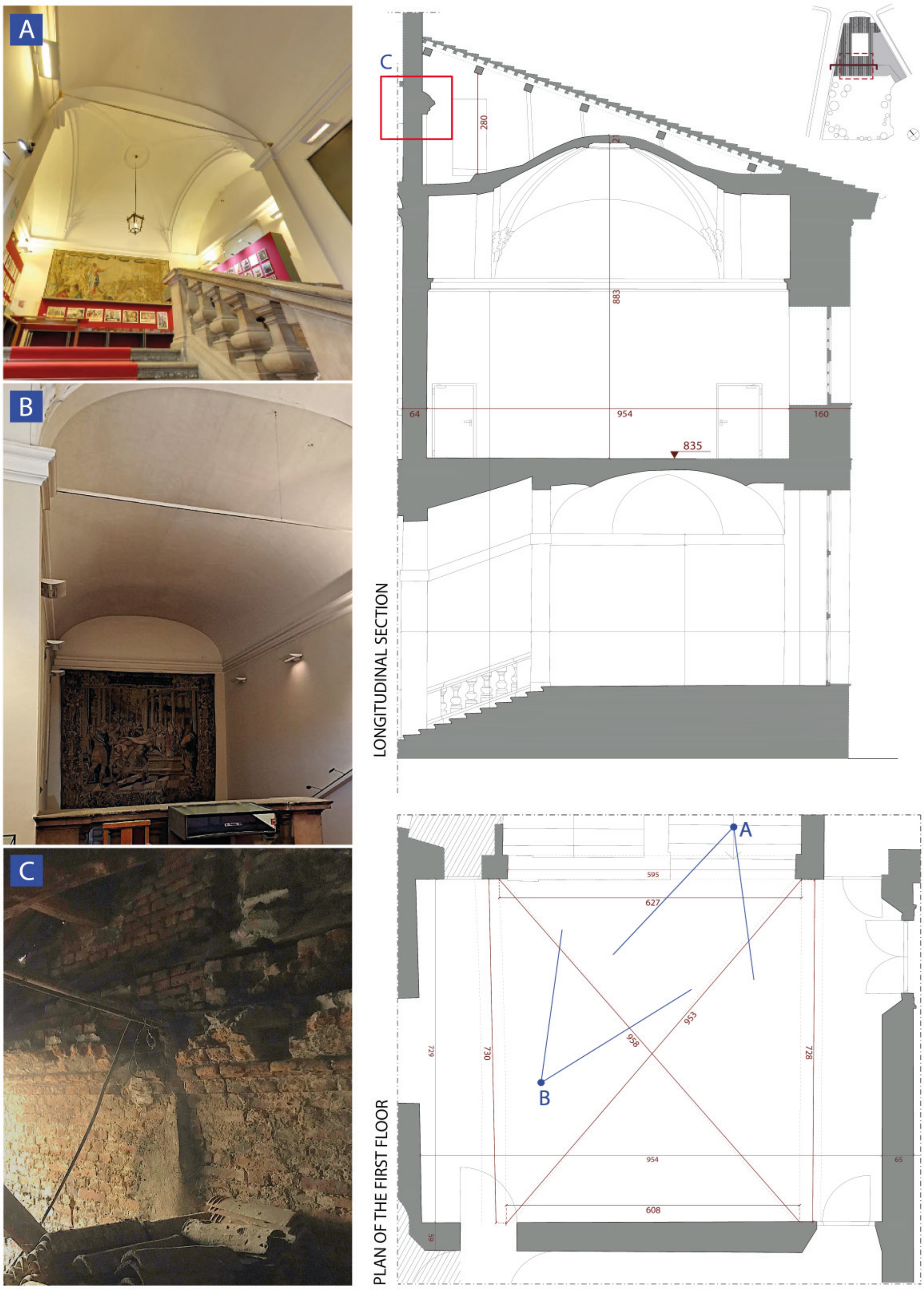


Fig. 3. Details of technical drawings of a first floor plan and longitudinal sections with photos (drawings and photos by the authors).

small entrance ramp give the room a certain grandeur, enhanced by the light coming from the large window facing the west, a place where once the Naviglio canal was.
The situation is different for the atrium on the first floor, reached by the monumental staircase that serves as an antechamber to the great halls. This room is covered by a large sail vault set on a rectangular base measuring 7.3 x 6 m. The presence of the pitched roof on the extrados probably allowed the vault construction to rise higher, adopting complete semicircular curvature and thus giving the room a larger open ambient (fig. 3).
The sail vault is accessible at the extrados, allowing it to be the subject of a careful study, both from the point of view of geometry and construction techniques. Additionally, the research provided a series of observations concerning the indications investigated in the historical treatises focusing on this particular type of internal roofing and ceiling solution.
The research of the ideological geometrical shapes and historical building techniques is essential for understanding before modelling [Capone, Lanzara 2019]. Among the treatises circulating in Milan in the 17th century, were *I quattro libri dell'architettura* by Palladio, published in Venice in 1570. Slightly after Ricchino's project for Palazzo Sormani, Guarino Guarini's manual called *Disegni di architettura civile ed ecclesiastica* was published in Turin in 1686. Another relevant work of Guarini on this topic is his posthumous work published in

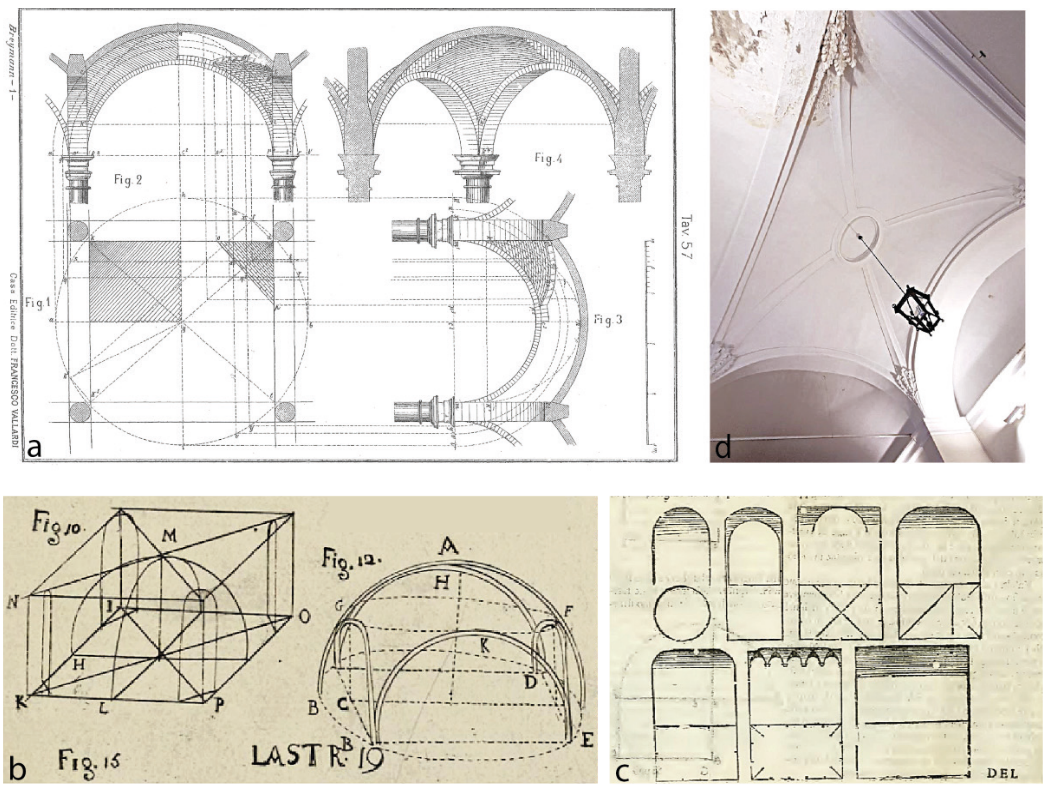


Fig. 4. Historical manuals studying the geometry of a sail vault. a: Breymann, *Archi, volte, cupole* (1853), p. 92; b: G. Guarini, *Disegni di architettura civile, opera postuma*, p.368 (1737); c: Palladio, *I quattro libri dell'architettura*, (1590) p.50.; d: Sail vault in Palazzo Sormani.

1737 [Guarini 1737, pp. 193-196]. These historical manuals are increasingly valuable today as a knowledge tool for retrospectively reconstructing the geometric genesis of the vaults designed by Ricchino. The drawings contained in the 19th-century manual by Breymann, originally published in 1853, provide a series of important indications related to the different geometries of vaults [Breymann 2003]. The volume *Stone Constructions and Wall Structures* gives particular attention to the construction techniques and brick arrangement [Breyman 1889] (fig. 4).
Regarding the sail vault on the first floor, from a geometrical stance, it is a spherical vault set on a rectangular plan, generated by the rotation of a semicircle around its centre. This type of vault began to be widely used in Lombard architecture from the 16th century

onwards, thanks to its concave and bulging shape, which allowed for light and airy spaces without the need for prominent supports or ribs, as visible in a cross vault. However, during the 17th century when the Baroque style flourished, there was an increased search for spectacular solutions even in domestic settings, where sail vaults were adopted in many noble residences and stately palaces, often enriched with frescoes and stucco decorative elements [Denti 1988, pp. 89-102] (fig. 5).
According to historical records, the construction of a sail vault started from the corners, where the pendentives were built first. Each row of bricks was placed to meet at the springing point, and then the wooden centring was set up. Its geometry was determined by considering that the intrados of a sail vault always forms a portion of a spherical surface whose centre is positioned along the vertical axis passing through the centre of the room (fig. 6). In the case of the studied vault, the centre of the sphere is not found at the impost line of the vault, but 120cm lower, along the central vertical axis. The height position of this centre had to be evalu-

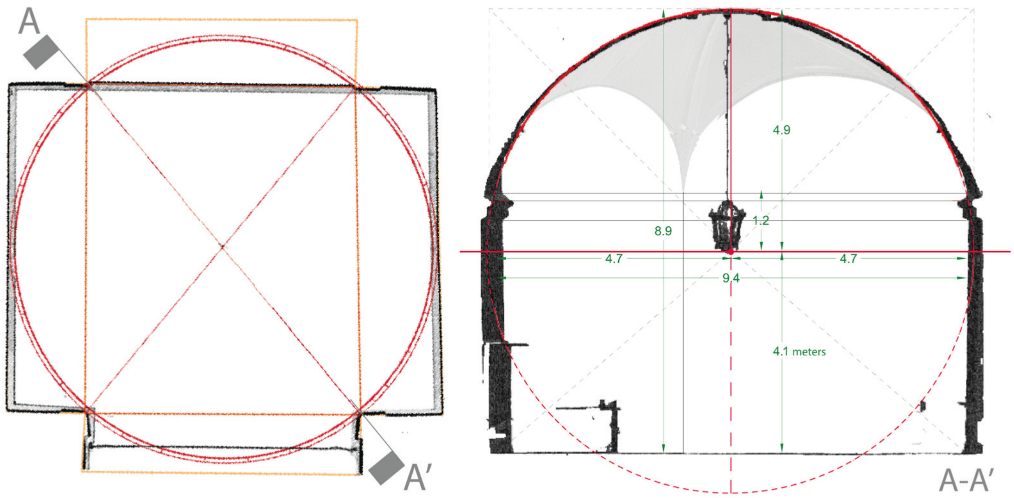


Fig. 5. Constructive genesis of the real vault starting from laser scanner data. The 3D-modelling approach combines technical drawings, gathered historical knowledge, and point cloud data (elaborations by the authors).

ated and calculated beforehand to determine the actual curve of the vault. However, this result was often reached empirically during construction phases by making progressive adjustments to the inclination of the courses to achieve the desired height at the crown. After the vault was closed, the centring was removed, leaving the structure free to settle. To prevent collapses, the haunches were reinforced immediately after the completion of the vault.
From a construction perspective, the vault has a thickness of approximately 21 cm at the top, with bricks arranged in a herringbone pattern laid on edge; only up to a height corresponding to the vault's springing, the thickness increases to two brick widths. This was done to enhance the vault's stability once the temporary centring was removed. This type of masonry technique allowed for minimal use of wooden ribs or, in some cases, eliminated the need for them.

Drawing and 3D modelling for the geometrical analysis of sail vaults

The sail vault is considered a simple architectural structure whose intrados belongs to a single geometric surface. It can be described as a hemisphere, or a dome, which is cut on four sides of the square or rectangular perimeter. Variations of the sail vault depend on construction techniques, the geometry of the room, and the shape of the parallel arches. It distributes weight evenly across its entire surface, creating a smoother transition over bearing walls without needing additional ribs [Cangi 2011].
The technical drawings of the plan and diagonal sections produced on the scale of 1:50 revealed initial irregularities in the rectangular base of the investigated vault. The section shows the highest

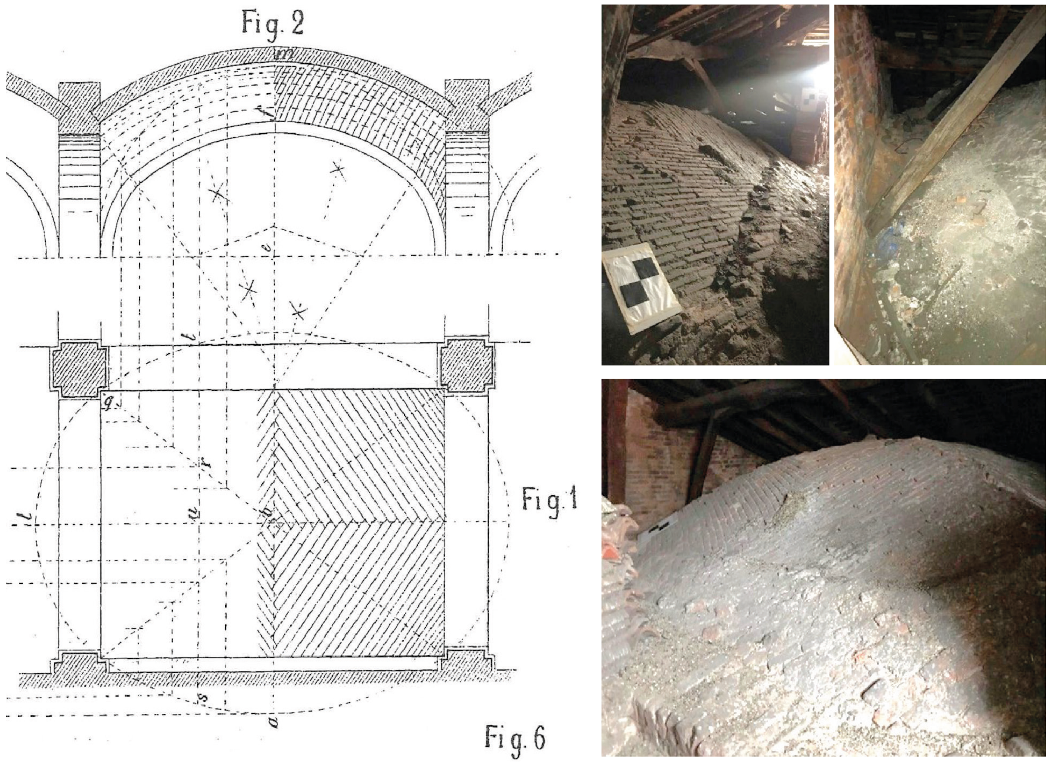


Fig. 6. Left: illustration of herringbone pattern from Breymann's manual (1853); right: photos of the extrados made by the authors.

point of 8.83 m, measured from the finished floor to the top of the crown. The geometrical shape and construction techniques necessary to build the sail vault were suitable solutions for the characteristics of the space. As a starting point for the research, the ideal sail vault was created using *Rhinoceros* 3D modelling software. The basic geometries derived from the laser scanning survey and two-dimensional drawings extracted the diagonal generating curve and formed the basis for the digital reconstruction of conceptual and real vaults. This approach helped identify potential irregularities in the actual structure [Antista, Morena, Mifsud 2024] (fig. 7).
The conceptual model of the sail vault was created starting from the rectangular base, whose measurements were obtained using the point cloud and drawing. The ideal intrados is produced by the sweeping method using one rail, meaning that the ideal circle, which spans from one to another corner of the diagonal section of the rectangular base, creates a

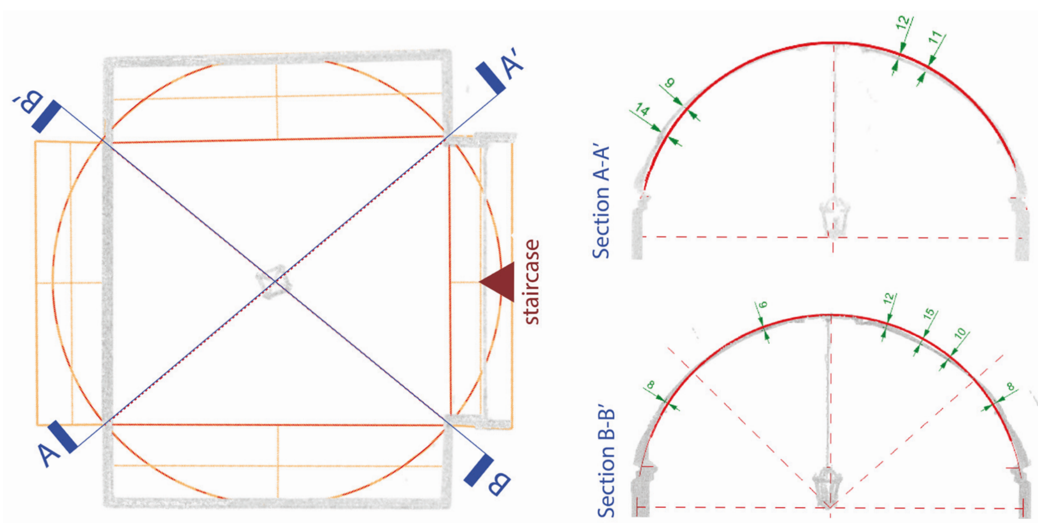


Fig. 7. The diagonal sections reveal the differences between regular and surveyed vaults (elaborations by the authors).

half curve that revolves following the path of the circumscribed circle in the plan. Studying diagonal sections, the parallel arches are lower by 33cm than the ideal vault surface. The result of the sweeping process is a dome, which subsequently has been cut by rectangular extrusions using Boolean difference. These extrusions originate from the parallel arches, which lean on the four sides of the rectangular base. The result is the archetype of a sail vault, which is created using the measurements of the real vault of Palazzo Sormani.
While the resulting shape closely matched the diagonal section, significant differences between the ideal and actual vault became apparent in the longitudinal and transverse

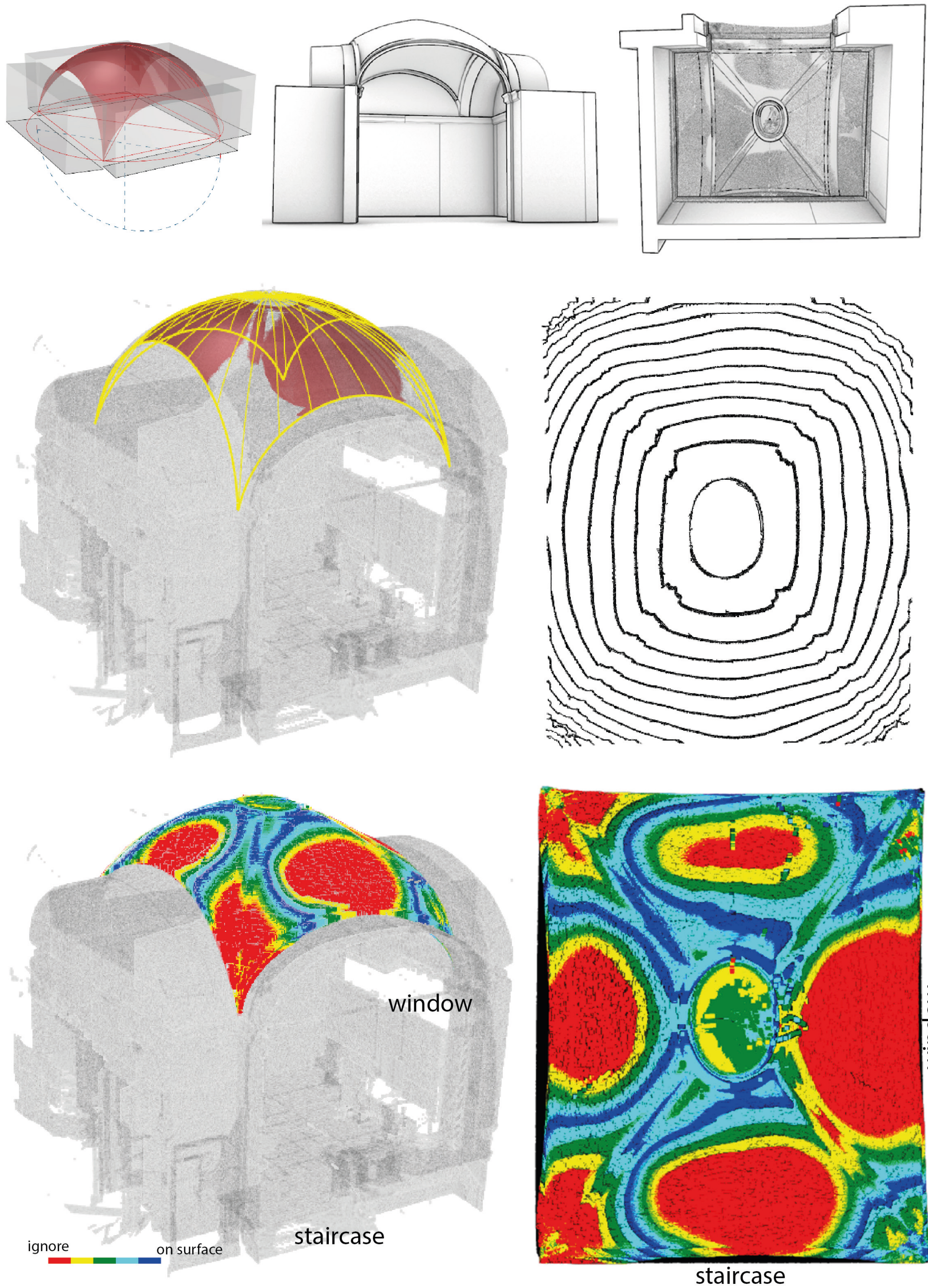


Fig. 8. After 3D modelling of the conceptual and real vaults in Rhinoceros software, point deviation and contour tools show the deformations and irregularities, especially pronounced in the direction of the staircase and window (elaborations by the authors).

cross-sections [Stanga 2020]. The diagonal section shows the irregular deviation, being more pronounced in the upper part near the flattened vault cap and the middle of the span of the vault, particularly toward the staircase and window. This has been analysed through the Point Deviation tool, where the ideal vault has been compared to the point cloud of the actual space (ranging from red for far points to blue for points on the surface). The changes in the actual surfaces can be caused principally by the unequal rectangular base, the shape of lowered arches, and the general movements of the building over time [Lopez-Mozo *et al*. 2022].
The second phase of the modelling involved the creation of a real geometry of the vault that has been modelled using the MeshFromPoints tool in Rhinoceros software, followed by the reduction and smoothing of the mesh polygons, so that it could be transformed into NURBS. The real vault has been sectioned using the Contour tool, which shows deformations of curves, shaping between a rectangle and a circle, which is caused by the form of the vault base. This could further indicate on almost symmetrical placement of deformations in the vault, being increased in the directions of the window and staircase [Brumana *et al.* 2013] (fig. 8).

Conclusion

The building's structures were severely affected by the bombing of August 1945, which caused the collapse of an entire wing. The study of the sail vault designed and built by Ricchino in 1650, still present in Palazzo Sormani, allowed the development of a methodology for investigating the condition and conservation state of vaulted elements in the Palace. Starting from historical sources and archival drawings, it was possible to trace the construction processes within which Ricchino's project for the new staircase took shape, integrating it into an existing building. The physical structure was surveyed in 3D using laser scanning, to produce two-dimensional drawings on a scale of 1:50. The study of historic architectural manuals allowed the data derived from the actual vault to be juxtaposed with the ideal shape of a sail vault which was generated according to guidelines of researched manuals within a digital environment [Capone *et al.* 2022].
This comparison was made possible through digital drawing and modelling tools, which served as cognitive investigative instruments capable of highlighting any areas of the vault characterised by geometric anomalies, deformations and bulging. Although these findings do not constitute an alarming statement about the structural condition, which must be verified with other mechanical and structural methods, they offer useful insights into the building's construction history.
The analysis revealed depressions in the vault structure at the mid-span of the supporting arches, precisely where the brick courses change direction and connect. This can be read in the point deviation map done in *Rhinoceros* software, but the hypothesis of an actual cause for this behaviour can be several. This can happen due to a normal settlement at the construction discontinuities (possibly occurring already during the removal of the centring), or it is caused by the long-term 'settling' over centuries. The third hypothesis is that the structure moved where it was the most sensitive near openings (window and staircase) during the shock wave of falling bombs.
The cross-reading of archive drawings, the indications provided by manuals, and three-dimensional survey data provided essential new geometric and constructive knowledge of these structures. New insights about vault structure have emerged, which is fundamental for its conservation.

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To cite this chapter: Daniela Oreni, Dina Jovanović (2025). Geometric and Constructive Analysis of Palazzo Sormani's Vaults through Drawings, Historical Manuals, and 3D Modelling. In L. Carlevaris *et al.* (Eds.). *èkphrasis. Descrizioni nello spazio della rappresentazione/èkphrasis. Descriptions in the space of representation.* Proceedings of the 46th International Conference of Representation Disciplines Teachers. Milano: FrancoAngeli, pp. 1681-1690. DOI: 10.3280/oa-1430-c842.

 ISBN 9788835182412